# 2D THERMO-MECHANICAL ANALYSIS OF ITAIPU BUTTRESS DAM USING THE FINITE ELEMENT METHOD IN FORTRAN

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Summary. The aim of the present work is to present a computational code in FORTRAN, based on the Finite Element Method, capable of simulating the thermal behavior of a two dimensional medium subjected to conduction heat transfer. The code is used to develop a numerical model of the buttress dam of the ITAIPU Hydroelectric Power Station (CHI). Thermo-structural coupling is performed using the temperature field as the nodal contour condition via the initial thermal deformation. In order to validate the model, the ANSYS® commercial software is used, the efficiency of which is proven in the technical literature, and the thermo-structural coupling of which is performed with the tools available in the Workbench. Finally, the results of the proposed coupling model are compared with the dam instrumentation data.

### 1 INTRODUCTION

Dams are hydraulic structures built transversely to the course of a river with the purpose of damming the water, with the intention of raising the water level for various purposes such as: irrigation, water supply, flood control, and power generation, among others. The safety of a dam is a permanent concern for governmental entities, both for the economic importance as for the risk of breaching, which involves, among other things, human lives, environmental impact and material losses (ELETROBRÁS, 2003).

As regards actions, the main loads present in a dam are: the weight of the dam itself, hydrostatic pressure, uplift, and internal forces due to thermal variation. The latter can affect the performance of the dam, reducing the strength and consequently the durability of the structure. Thus, the precise assessment of the temperature field is important to determining the location of stresses of thermal origin (ZHU BOFANG, 2004).

During the dam construction phase, most of the internal thermal variation of the structure occurs due to the chemical reactions of the concrete. Subsequently, this heat source may be disregarded, and the temperature variation becomes essentially seasonal, where the thermal amplitude occurs due to the heat flux between the dam surface and the environment (HICKMANN, 2016).

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In order to estimate the temperature field, as well as the stresses and displacements of thermal origin, it is necessary to solve differential equations that represent these physical phenomena. Instrumental monitoring data can be used to determine the boundary conditions, and thus, find a solution to the problem. The exact solution is always the ideal, based on differentiable algebraic methods, but, depending on complexity, it becomes impractical. For this reason, numerical methods are used to obtain approximate solutions. Among the several methods, the Finite Element Method is one of the most popular in structural engineering analyzes (HICKMANN, 2016).

# 2 HEAT TRANSFER EQUATION

According to Incropera (1998), heat is defined as the form of energy that is transferred from one system to another due to the temperature difference between them. When there is a steady-state temperature gradient - that is, one without time variations - in a solid or fluid medium, conduction is considered to refer to heat transfer. The heat transfer rate equation is known as the Fourier law. Considering an isotropic and homogeneous material, after applying simplifications, the governing equation of heat transfer in a two-dimensional medium can be written according to Eq.(1).

$$k\left(\frac{\partial^2 \mathbf{T}}{\partial x^2} + \frac{\partial^2 \mathbf{T}}{\partial y^2}\right) + Q = 0 \tag{1}$$

where k is the coefficient of thermal conductivity, Q the heat source and T the temperature.

# 3 STRAINS IN SOLIDS DUE TO TEMPERATURE VARIATION

The stress and strain are related through the constitutive equation, presented in Eq.(2). The simplest form of the constitutive equation is that of linear elasticity, which is a generalization of Hooke's Law.

$$\sigma = C\epsilon \tag{2}$$

where  $\sigma$  is the stress,  $\mathcal{C}$  the elastic constant and  $\epsilon$  the strain.

If the material is isotropic, the properties are symmetrical with respect to the three planes. In this case, only two independent constants are required to define the fundamental elasticity equations. These constants are the Young's Modulus (E) and the Poisson ratio (v).

Thus, the constitutive equation of an isotropic material in the plane strain state is:

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} = \frac{E}{(1+v)(1-2v)} \begin{pmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & \frac{1-v}{2} \end{pmatrix} \begin{pmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{pmatrix}$$
 (3)

where  $\sigma_x$ ,  $\sigma_y$ ,  $\tau_{xy}$  are the x and y direction stresses and the shear stress, respectively; E is the elasticity modulus and v is the Poisson ratio, and  $\epsilon_x$ ,  $\epsilon_y$ ,  $\gamma_{xy}$  are the x and y direction strains and the distortional strain, respectively.

In a free body, the increase of temperature ( $\Delta T$ ) results in a strain that depends on the expansion thermal coefficient ( $\alpha$ ) of the material (the temperature change does not cause shear strains). Thus, the initial strain vector due to temperature variation can be stated according to Eq.(4).

$$\epsilon_{0} = \begin{pmatrix} \epsilon_{x} \\ \epsilon_{y} \\ \epsilon_{z} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{pmatrix} = \begin{pmatrix} \alpha \Delta T \\ \alpha \Delta T \\ \alpha \Delta T \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

where  $\epsilon_0$  is the initial strain of the material due to temperature variation,  $\alpha$  is the coefficient of thermal expansion,  $\Delta T$  is the temperature variation, and  $\tau_{xy}$ ,  $\tau_{yz}$ ,  $\tau_{zx}$  are the shear strains.

# 3 THREE-NODES TRIANGULAR FINITE ELEMENT

According to ASGHAR (2005), a triangular element is simple and versatile for solving two-dimensional problems. Almost all forms can be discretized using this type of element. At Figure 1 a triangular element with degrees of freedom is presented for solving the thermal-structural problem. Each node has one degree of freedom of temperature and two degrees of freedom for displacements (x and y).

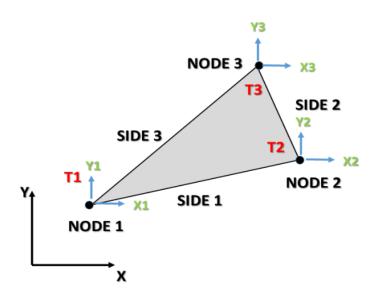


Figure 1. Three-Node Triangular Element Source: ASGHAR (2005)

The coordinates of nodes 1, 2 and 3 are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ , respectively. The error associated with the discretization of a contour curve through several straight lines represented by the edges of the element can be reduced by increasing the number of elements in that region.

## 3.1. FEM IN 2D TEMPERATURE FIELD RESOLUTION

In the case of the temperature field problem, each node has one degree of freedom, the temperature. Considering an edge temperature as a boundary condition and that the model has no heat source, the equation of the element takes the form presented in Eq.(5).

$$k_k T = 0 (5)$$

where T is the nodal temperature vector.

The heat conduction matrix is,

$$k_k = \iint\limits_A BCB^T dA = ABCB^T \tag{6}$$

Considering a three-node triangular element, the stiffness matrix is presented according to Eq.(7),

$$k_{k} = \frac{1}{4A} \begin{pmatrix} k_{x}b_{1}^{2} + k_{y}c_{1}^{2} & k_{x}b_{1}b_{2} + k_{y}c_{1}c_{2} & k_{x}b_{1}b_{3} + k_{y}c_{1}c_{3} \\ k_{x}b_{1}b_{2} + k_{y}c_{1}c_{2} & k_{x}b_{2}^{2} + k_{y}c_{2}^{2} & k_{x}b_{2}b_{3} + k_{y}c_{2}c_{3} \\ k_{x}b_{1}b_{3} + k_{y}c_{1}c_{3} & k_{x}b_{2}b_{3} + k_{y}c_{2}c_{3} & k_{x}b_{3}^{2} + k_{y}c_{3}^{2} \end{pmatrix}$$
(7)

where  $b_1$ ,  $b_2$  and  $b_3$  are variables related to the nodal coordinates of the element and  $k_x$  and  $k_y$  are the coefficients of thermal conductivity of the material in the x and y directions, respectively.

#### 3.2. THERMAL-MECHANICAL COUPLING WITH THE FEM

The formulation of the finite element method for the thermal-mechanical problem, considering only the initial strain due to the variation of temperature, is presented in Eq.(8),

$$K d = r_q + r_b + r_\epsilon \tag{8}$$

where K is the stiffness matrix, presented in Eq.(9),

$$k = \iint_{A} BCB^{T} dA \tag{9}$$

and

$$B^{T} = \frac{1}{2A} \begin{pmatrix} b_{1} & 0 & b_{2} & 0 & b_{3} & 0 \\ 0 & c_{1} & 0 & c_{2} & 0 & c_{3} \\ c_{1} & b_{1} & c_{2} & b_{2} & c_{3} & b_{3} \end{pmatrix}$$
(10)

is the transformation matrix.

For the two-dimensional strain state,

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$$C = \frac{E}{(1+v)(1-2v)} \begin{pmatrix} 1-v & v & 0\\ v & 1-v & 0\\ 0 & 0 & \frac{1-v}{2} \end{pmatrix}$$
(11)

and  $r_{\epsilon}$  is the vector of equivalent load due to the initial strains originated by the variation of body temperature, according to Eq.(12).

$$r_{\epsilon} = h \iint_{A} BC \epsilon_{0} dA \tag{12}$$

where  $\epsilon_0 = (1 + v)(\alpha \Delta T \ \alpha \Delta T \ 0)^T$  (for the two-dimensional strain state) and h is the unit thickness of the element.

#### 4 THERMO-MECHANICAL ANALYSIS OF THE ITAIPU BUTTRESS DAM

The ITAIPU Hydroelectric Power Station (CHI), built between 1974 and 1982, located on the Paraná River, near the city of Foz do Iguaçu in the state of Paraná, is a dam formed by a set of sections, composed by earthfill, rockfill, massive gravity, hollow gravity and buttress dams.

The dam is divided into the main dam, the diversion structure, the right bank earthfill dam, the left bank rockfill and the earthfill dam. The section E is a transition dam, located on the right abutment, between the main dam (section F) and the right side dam (section D). It consists of 6 buttress blocks, where E-6, a key block, is fully instrumented. At **Figure 2** the lateral view of this block is presented (ITAIPU BINATIONAL, 2009).

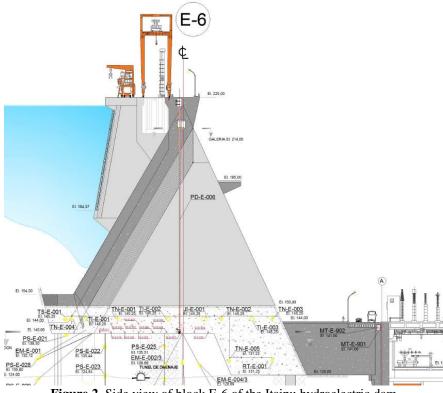


Figure 2. Side view of block E-6 of the Itaipu hydroelectric dam

The main Itaipu dam monitoring instruments used in this work are:

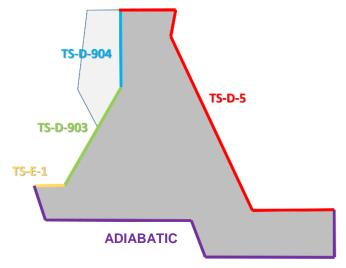
- Direct plumbline;
- Thermometers that measure the temperature of the surface and the interior of the dam.

In relation to the properties of the concrete, the following values were adopted:

Property	Adopted value	Source
Modulus of elasticity (E)	35 GPa	PROMON
Poisson ratio	0.16	NBR6118
Thermal conductivity (k)	1.8492	<b>PROMON</b>
Thermal dilation coefficient ( $\alpha$ )	0.0001 /°C	NBR6118

#### 4.1. 2D GEOMETRIC MODEL OF THE BUTTRESS BLOCK

Based on the general drawings and the location of the dam thermometers, the model for thermo-mechanical analysis is presented in **Figure 3**.



**Figure 3.** Mathematical model with boundary conditions extrapolated.

The TS-D-904, TS-D-903, TS-D5 and TS-E-1 are the surface thermometers installed in block E-6 and other nearby blocks.

In Table 1 the winter and summer temperature values used as contour conditions are presented.

Thermometer	Winter (°C)	Summer (°C)
	Date 12/07/2010	Date: 11/01/2010
TS-D5	18.46	32.33
TS-D-904	21.1	29.27
TS-D-903	20.76	28.62
TS-E-1	21.47	22.65

Table 1: Summer and winter temperatures

The mesh was created based on the general design of the block E-6. In order to determine the nodal coordinates, the *AUTOCAD* software was used. The mesh is formed only by three-node triangular elements, according to the element described in **Section 3.** A regular and uniform mesh was preserved to facilitate graphical plotting of the results of the temperature field and displacements through using the *GNUPLOT* software. In total, the mesh has 321 nodes and 553 elements, according to **Figure 4**.

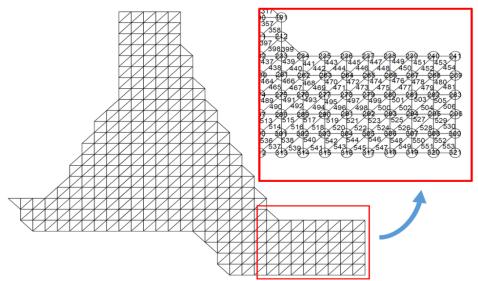


Figure 4. Dam grid with triangular elements

#### 4.2. ALGORITHM IN FORTRAN

The algorithm was developed in the programming language FORTRAN 95 through the software CODEBLOCKS 16.1, an open source programming platform. For the graphic plotting of the results the *GNUPLOT* software was used, this being a package of multiple platform graphics that works via command line.

At **Figure 5** the general schematic diagram of the algorithm and the programs that were used in each phase are presented. ANALYZE

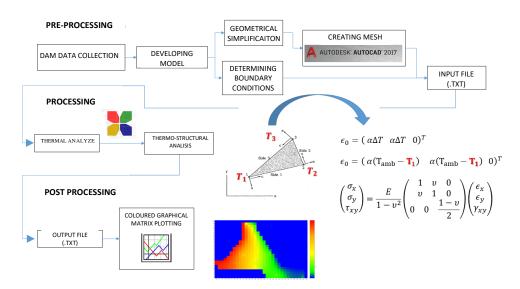


Figure 5. Schematic general diagram of the algorithm

#### 5. RESULTS

The numerical results obtained using the developed code, considering the temperatures presented in *Table 1*, were compared with the results obtained with ANSYS®, to validate the proposed coupling model.

Considering the boundary conditions, such as summer temperatures, the results of the fields of temperature and horizontal displacement are presented in **Figure 6**, **Figure 7**, **Figure 8** and **Figure 9**.

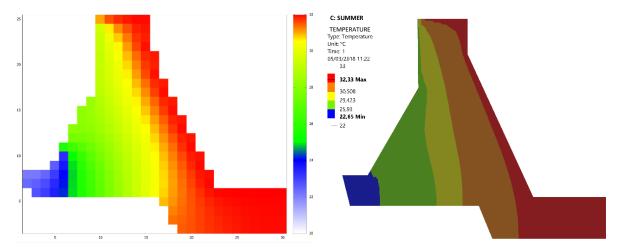


Figure 6. Algorithm temperature field

Figure 7. ANSYS® temperature field

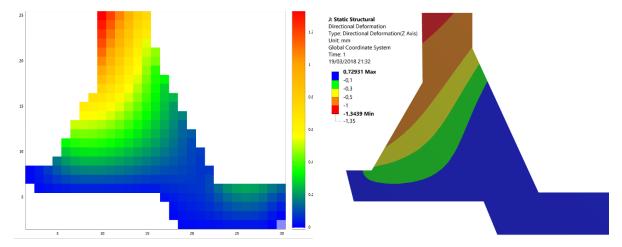


Figure 8. Algorithm horizontal displacement field

Figure 9. ANSYS® horizontal displacement field

Considering the contour conditions measured in winter, the results of the temperature field and horizontal displacement are presented in **Figure 10**, **Figure 11**, **Figure 12** and **Figure 13**.

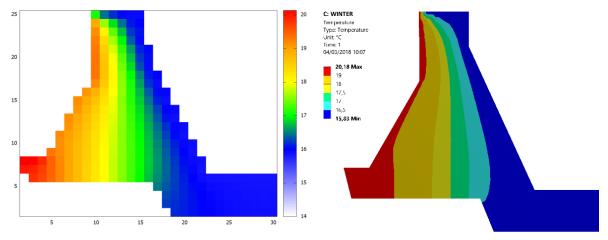


Figure 10. Algorithm temperature field

Figure 11. ANSYS® temperature field

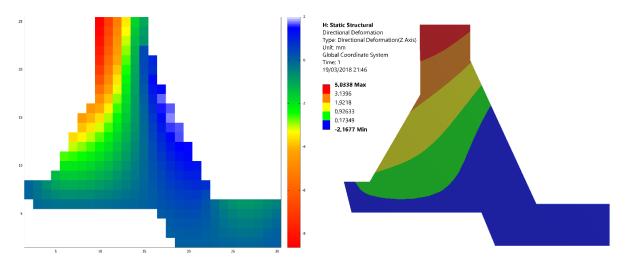


Figure 12. Algorithm horizontal displacement field

Figure 13. ANSYS® horizontal displacement field

# 6. CONCLUSIONS

As a first conclusion, it can be noted an excellent accordance between the results obtained from the developed code and the ANSYS® software.

In relation to the thermal problem, the greatest difference in nodal temperature found between the model solved by the algorithm and the model of the Ansys® program is approximately 0.5 °C. Regarding the structure displacements due to temperature variation, the model presented the same physical behavior and values close to the values of the Ansys® model. Considering the stead-state heat transfer, the models showed maximum and minimum horizontal displacements within the predicted design values.

According to internal thermometers, the core of the structure exhibits temperature below surface temperature during the summer, and temperatures above surface temperatures during the winter. This is a strong indication of a transient heat transfer behavior inside the dam. A new temperature analysis varying over time should be studied for refinement and enhancement of the algorithm in the resolution of bulk concrete structures.

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#### REFERENCES

ALVES, L. M. Métodos dos Elementos Finitos - Apostila organizada das aulas de métodos dos elementos finitos do curso de doutorado. Universidade Federal Do Paraná. 2007. Available at: <a href="http://www.portalsaberlivre.com.br/manager/uploads/apostilas/1316558246">http://www.portalsaberlivre.com.br/manager/uploads/apostilas/1316558246</a>. pdf>. Accessed on: November 3, 2017.

ASGHAR, B. M. Fundamental element analysis and applications. Hoboken: John Wiley & Sons. 2005.

BRAGA, W. F. Transmissão de calor. São Paulo: Pioneer Thomson Learning, 2004.

HICKMANN, T. Análise do efeito da variação térmica sazonal em barragem de contrafortes. Thesis (PhD in Numerical Methods in Engineering) - Universidade Federal do Paraná, Curtiba. 2016.

HETNARSKI, R., ESLAMI, M., Thermal stresses - Advanced theory and applications. Springer. 2009.

INCROPERA, FRANK P.; DEWITT, DAVID P. Fundamentos de transferência de calor e massa. 4ª Edição. LTC Editora. 1998

ITAIPU BINACIONAL. Itaipu Hydroelectric Power Plant - Engineering aspects. Technical Directorate. Foz do Iguaçu: ITAIPU BINACIONAL, 2009.

ITAIPU BINACIONAL. SAT- Sistema de arquivos técnicos 2017.

JAIME G. & MOLINA P. Fundamentos del método de elemento finito. Primera Edición. Ingeniería Mecánica Universidad Mayor de San Andrés (UMSA). 2010.

KIM, N. H. SANKAR, B. V. Introdução à análise e ao projeto em elementos finitos. LTC Editora, 2011.

LÉGER, P., SEYDOU, S. Seasonal thermal displacements of gravity dams located in Northern regions. Journal of Performance of Constructed Facilities volume 23. 2009.

LOGAN, D. L. A First course in the finite element method. Ed. CL-Engineering, 4th edition, 2006.

SA, E. W. E. B. Critérios de projeto civil de usinas hidrelétricas. Rio de Janeiro, 2003.

SOUZA, R. M. O método dos elementos finitos aplicado ao problema de condução de calor. Universidade Federal Do Pará, Belém, 2003. Available at: <a href="http://www.ufpa.br/nicae/integrantes/remo\_souza/TrabPublicados/Apostilas/ApostilaElementosFinitosNiCAE.pdf">http://www.ufpa.br/nicae/integrantes/remo\_souza/TrabPublicados/Apostilas/ApostilaElementosFinitosNiCAE.pdf</a>. Accessed on: November 3, 2017.

TATIN, M., BRIFFAUT, M., DUFOUR, F., SIMON A., FABRE J. P. Thermal displacements of concrete dams: accounting for water temperature in statistical models. Engineering Structures. VOLUME 91. EISEVIER. 2015). P.26-39. Available

ZHU BUFANG. Thermal stresses and temperature control of mass concrete. Elsevier. First Edition. China Institute of Water Resources and Hydropower Research and Chinese Academy of Engineering, 2004.

WELTY, J. R.; WICKS, C. E.; WILSON, R. E.; RORRER, G. L. Fundamentals of momentum, heat and mass transfer, 4th ed., Hoboken (NJ): John Wiley & Sons, 2001.